Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

EQUATION OF THE MEAN MONTHLY 21-YEAR TEMPERATURE CURVE OF LAWRENCE, KANSAS.

BY E. C. MURPHY, LAWRENCE.

Those who have examined Part I, Vol. 12, of the transactions of the Kansas Academy of Science, have seen on the last page a curve called, "Curve of Mean Daily Temperatures for Twenty-one Years."

In order to show clearly the meaning of the equation that I have found, I will explain briefly how this curve is gotten.

Temperature observations have been taken at Lawrence by Professor F. H. Snow three times each day for twenty-one years. The mean of the three temperatures for any one day gives the daily mean temperature for that day. The sum of the daily mean temperatures of any day for twenty-one years divided by twenty-one, gives the daily mean temperature for that day for the twenty-one years. These temperatures are platted to some scale, using time as an absissa and temperature as an ordinate, giving a series of points; these points are connected by a smooth curve. This curve is called curve of *daily* mean temperatures.

If now we add these daily mean temperatures for any month and divide by the number of days in the month, we get the mean monthly temperature of this month for twenty-one years. If these temperatures be platted as above, and the points connected by straight lines, we get chords of the curve whose equation I have found. It is an equation, or relation between mean monthly temperature and time, which has the twelve mean monthly values of temperature for values of time reckoned from the middle of January to the middle ef each succeeding month.

Temperature is a periodic function of time, the period being one year. We have twelve values of this function, and the corresponding values of the time to find the form of the function, or the equation of the curve.

If $T =$ temperature, and $t =$ time, we have

$T = \phi(t)$; or if θ is any angle which varies from 0 to 360°, as t varies from 0 to 365 days, we have

$$T = f(\theta) \ldots\ldots\ldots\ldots\ldots (1)$$

The general form of a periodic function is

$$\left.\begin{array}{l} f(\theta) = A + a_1 \cos.\ \theta + a_2 \cos.\ 2\,\theta + \\ \qquad + b_1 \sin.\ \theta + b_2 \sin.\ 2\,\theta + \end{array}\right\} \ldots\ldots\ldots\ldots\ldots (2)$$

in which A, a_1, a_2, b_1, b_2, etc., are constants.

In this case, since there are only twelve observed temperatures, equation (2) reduces to

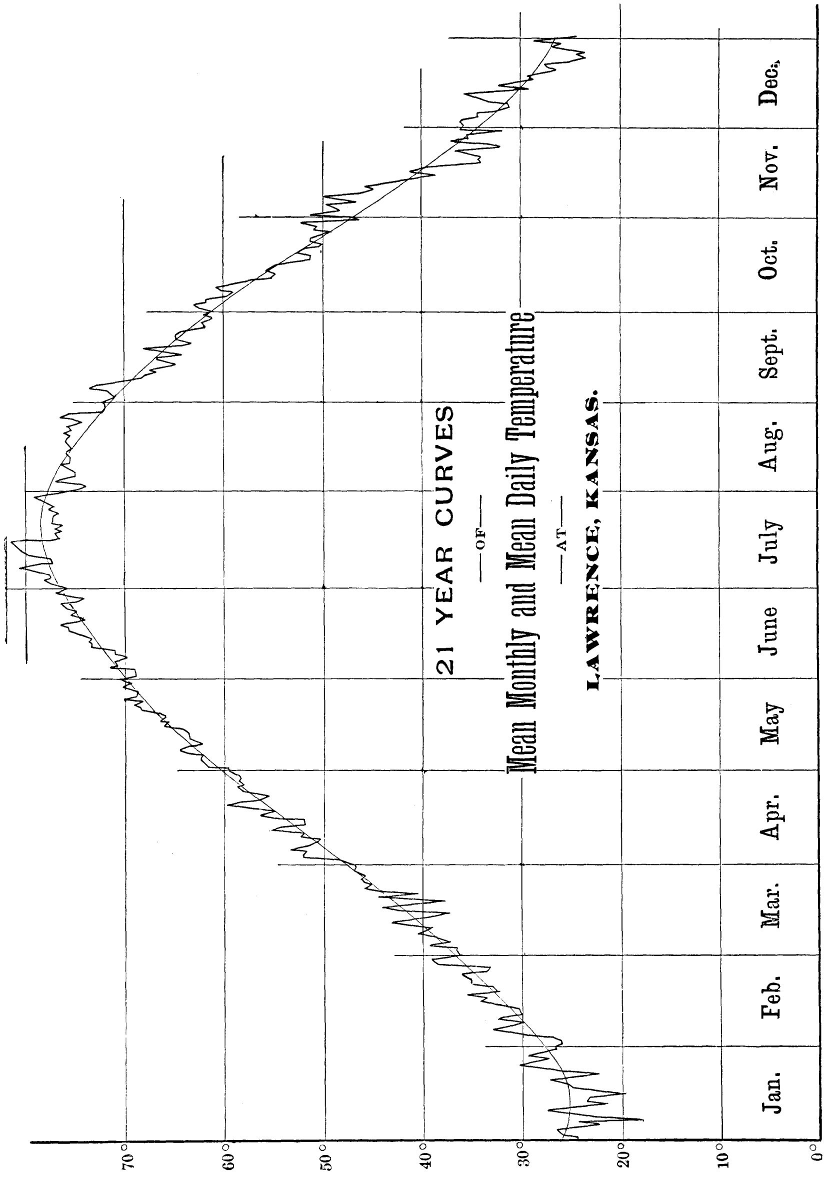
$$\left.\begin{array}{l} f(\theta) = A + a_1 \cos.\ \theta + a_2 \cos.\ 2\,\theta + a_3 \cos.\ 3\,\theta + a_4 \cos.\ 4\,\theta + a_5 \cos.\ 5\,\theta \\ \qquad + b_1 \sin.\ \theta + b_2 \sin.\ 2\,\theta + b_3 \sin.\ 3\,\theta + b_4 \sin.\ 4\,\theta + b_5 \sin.\ 5\,\theta \end{array}\right\} (3)$$

In (3) there are eleven terms and eleven constants.

The twelve observed temperatures, or values of $f(\theta)$, give twelve equations, from which the values of the eleven constants are to be found.

These twelve equations, called observation equations, are

$$\left.\begin{array}{l} T_1 = A + a_1 \cos.\ (0) + a_2 \cos.\ 2\ (0) + \ldots\ldots + a_5 \cos.\ 5\ (0) \\ \qquad + b_1 \sin.\ (0) + b_2 \sin.\ 2\ (0) + \ldots\ldots + b_5 \sin.\ 5\ (0) \\ T_2 = A + a_1 \cos.\ 30^\circ + a_2 \cos.\ 2\ (30^\circ) + \ldots\ldots + a_5 \cos.\ 5\ (30^\circ) \\ \qquad + b_1 \sin.\ 30^\circ + b_2 \sin.\ 2\ (30^\circ) + \ldots\ldots + b_5 \sin.\ 5\ (30^\circ) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ T_{12} = A + a_1 \cos.\ (330^\circ) + a_2 \cos.\ 2\ (330^\circ) + \ldots\ldots + a_5 \cos.\ 5\ (330^\circ) \\ \qquad + b_1 \sin.\ (330^\circ) + b_2 \sin.\ 2\ (330^\circ) + \ldots\ldots + b_5 \sin.\ 5\ (330^\circ) \end{array}\right\} \ldots\ldots (4)$$



21 YEAR CURVES

—OF—

Mean Monthly and Mean Daily Temperature

—AT—

LAWRENCE, KANSAS.

If we substitute $A = \frac{\Sigma T}{n} + B$, $T_1 - \frac{\Sigma T}{n} = l_1$, $T_2 - \frac{\Sigma T}{n} = l_2$, etc., in (4), they reduce to the more convenient form:

$$\left.\begin{array}{l}
\left.\begin{array}{l} B + a_1 \cos. (0) + a_2 \cos. 2\,(0) + \ldots. + a_5 \cos. 5\,(0) \\ \quad + b_1 \sin. (0) + b_2 \sin. 2\,(0) + \ldots. + b_5 \sin. 5\,(0) \end{array}\right\} + l_1 = \surd_1 \\
\left.\begin{array}{l} B + a_1 \cos. (30°) + a_2 \cos. 2\,(30°) + \ldots. + a_5 \cos. 5\,(30°) \\ \quad + b_1 \sin. (30°) + b_2 \sin. 2\,(30°) + \ldots. + b_5 \sin. 5\,(30°) \end{array}\right\} + l_2 = \surd_2 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\left.\begin{array}{l} B + a_1 \cos. (330°) + a_2 \cos. 2\,(330°) + \ldots. + a_5 \cos. 5\,(330°) \\ \quad + b_1 \sin. (330°) + b_2 \sin. 2\,(330°) + \ldots. + b_5 \sin. 5\,(330°) \end{array}\right\} + l_{12} = \surd_{12}
\end{array}\right\} \ldots (5)$$

There being twelve observation equations, and only eleven constants to be determined, we can find their most probable values. The most probable values of the constants are those which make $\Sigma \surd^2$ a minimum, $\surd$ being any residual error.

The eleven normal equations, from which the values of the eleven constants are found, are gotten by placing the first differential coëfficient of $\Sigma \surd^2$ equal to zero.

These normal equations, after simplification, are as follows:

$$\left.\begin{array}{l}
12B = \Sigma l \\
6a_1 = l_1 - l_7 + (l_3 - l_9 - l_5 + l_{11}) \sin. 30° + (l_2 - l_8 - l_6 + l_{12}) \cos. 30° \\
6b_1 = l_4 - l_{10} + (l_3 - l_9 + l_5 - l_{11}) \cos. 30° + (l_2 - l_8 + l_6 - l_{12}) \sin. 30° \\
6a_2 = l_1 + l_7 - l_{10} - l_4 + (l_2 + l_8 + l_6 + l_{12} - l_3 - l_9 - l_5 - l_{11}) \sin. 30° \\
6b_2 = (l_2 + l_8 - l_6 - l_{12} + l_3 - l_9 - l_5 - l_{11}) \cos. 30° \\
6a_3 = l_1 + l_5 + l_9 - (l_3 + l_7 + l_{11}) \\
6b_3 = l_2 + l_6 + l_{10} - (l_4 + l_8 + l_{12}) \\
6a_4 = l_1 + l_7 + l_4 + l_{10} - (l_2 + l_8 + l_3 + l_9 + l_5 + l_{11} + l_6 + l_{12}) \sin. 30° \\
6b_4 = (l_2 - l_3 + l_5 - l_6 + l_8 - l_9 + l_{11} - l_{12}) \cos. 30° \\
6a_5 = l_1 - l_7 + (-l_2 + l_8 + l_6 - l_{12}) \cos. 30° + (l_3 - l_9 - l_6 + l_{11}) \sin. 30° \\
6b_5 = l_4 - l_{10} + (l_{11} - l_3 + l_9 - l_5) \cos. 30° + (l_2 + l_6 - l_8 - l_{12}) \sin. 30°
\end{array}\right\} \ldots (6)$$

The following table gives the mean monthly temperatures or values of T (computed from Prof. Snow's meteorological record), and the values of $T - \frac{\Sigma T}{n} = l$:

	T.	*l.*		T.	*l.*		T.	*l.*
January......	24.86°	—28.08	May..........	65.31°	+12.38	September...	66.38°	+13.45
February.....	32.05	—20.88	June..........	73.62	+20.69	October......	54.43	+1.50
March..........	41.47	—11.46	July...........	78.07	+25.14	November...	39.86	—13.07
April............	54.18	+1.25	August.......	75.44	+22.51	December...	29.46	—23.47

Substituting the value of l_1, l_2, l_3, etc., in equations (6), and solving, we have $A = 52.93°$, $a_1 = -25.70$, $a_2 = -1.12$, $a_3 = -0.47$, $a_4 = -0.05$, $a_5 = -0.43$, $b_1 = +0.10$, $b_2 = 1.02$, $b_3 = 0.17$, $b_4 = 0.25$, and $b_5 = -0.06$.

Substituting these values in equation (3) we have

$$\left.\begin{array}{l}
T = f(\theta) = 52.93 - 25.70 \cos. \theta - 1.15 \cos. 2\theta - 0.47 \cos. 3\theta - 0.05 \cos. 4\theta - 0.43 \\
\qquad\qquad \cos. 5\theta \\
\qquad + 0.10 \sin. \theta + 1.02 \sin. 2\theta + 0.17 \sin. 3\theta + 0.25 \sin. 4\theta + 0.01 \\
\qquad\qquad \sin. 5\theta
\end{array}\right\}.$$

Note.—In the simplification of the normal equations we use the following results, easily proven true:

$\Sigma \cos. m\theta = 0$, $\Sigma \sin. m\theta = 0$, $\Sigma \sin. m\theta \cos. m\theta = 0$, $\Sigma \cos.^2 m\theta = \frac{n}{2}$, and $\Sigma \sin.^2 m\theta = \frac{n}{2}$, m taking all values from 0 to $n - 1$ and $\theta = \frac{360}{n}$, n being any integer.